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RELATIVISTIC CALCULATION OF POLARIZATION OBSERVABLES IN $NN \to d\pi$ PROCESSES

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Abstract

A detailed analysis of processes of the type $NN \to d\pi$ is presented taking into account the exchange graphs of a nucleon and a pion. A large sensitivity of polarization observables to the off-mass shell effects of nucleons inside the deuteron is shown. Some of these polarization characteristics can change the sign by including these effects. The influence of the inclusion of a P-wave in the deuteron wave function is studied, too. The comparison of the calculation results of all the observables with the experimental data on the reaction $pp \to d\pi^+$ is presented.

Key-words: polarization observables, off-mass shell effects, deuteron wave function.

1 Introduction

As known, pion production in NN collisions, in particular the channel $NN \to d\pi$, has been investigated by many theorists and experimentalists over the last decades. An earlier study of this reaction [1] shows that the excitation of the Δ-isobar is a crucial ingredient for explaining the observed energy dependence of the cross section. A lot of papers are based on multichannel Schrödinger equations with separable or local potentials [2]. However, those studies were performed within the nonrelativistic approach. Early attempts to develop the relativistic approach were made in [3]. Both the pole graph, i.e. one-nucleon exchange, and the rescattering graph presented below were calculated in those papers. As shown, this diagram should result in a dominant contribution to the cross section of the discussed process. By the calculation of this one, some approximations, in particular the factorization of nuclear matrix elements, neglect of recoil etc., were introduced which lead to an uncertainty of the final results. A more careful relativistic study of the reaction $pp \to d\pi^+$ was made in [4]. The pole and rescattering graphs were shown to be insufficient to describe the experimental data; higher order rescattering contributions should be taken into account. However, in this approach there was no successful description of all the polarization observables, especially the asymmetries A_{y0}, iT_{11}. Really, analyzing reactions of the type $NN \to d\pi$, there occurs a problem related to the off-mass shell effects of nucleons inside the deuteron. When the pion is absorbed by a two-nucleon pair or the deuteron, the pion energy is shared between two nucleons. So, for example, the relative momentum of the nucleon inside the deuteron increases at least by a value $\sim \sqrt{m\mu} = 360 MeV$ if the rest pion is absorbed by the off-shell nucleon what corresponds to intra-deuteron distances of the order of $\sim 1/\sqrt{m\mu} \simeq 0.6 fm$. This means that the absorption

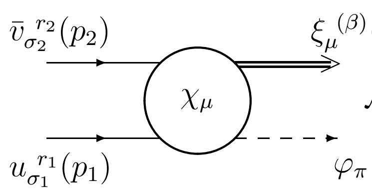
process should be sensitive to the dynamics of the πNN system at small distances. In this paper we concentrate mainly on the investigation of the role of these effects and the contribution of the P-wave of the deuteron wave function [6]. The sensitivity of all the polarization observables to these effects is studied, and it is shown that some polarization characteristics can change the sign by including the off-mass shell effects of nucleons inside the deuteron.

The detailed covariant formalism of the construction of the relativistic invariant amplitude of the reaction $NN \to d\pi$ for this process are presented in chapter 2. We analyze in detail both the pole graph, one-nucleon exchange, and the triangle diagram, i.e. the pion rescattering graph, in sections 3. The inputs by this consideration, the covariant pseudoscalar πNN and deuteron $d \to pn$ vertices, are discussed in detail. The discussions of the obtained results and the comparison with the experimental data are presented in chapter 4. The conclusion is presented in the last section 5.

2 General Formalism

• *Relativistic invariant expansion of the amplitude.*

We start with the basic relativistic expansion of the reaction amplitude $NN \to d\pi$ using Itzykson-Zuber conventions [7]. In the general case, the relativistic amplitude of the production of two particles of spins 1 and 0 by the interaction of two spin 1/2 particles has 6 relativistic invariant amplitudes if all particles are on-mass shell and taking P-invariance into account. It can be written in the following form:



$$\mathcal{M}^{\beta}_{\sigma_2,\sigma_1}(s,t,u) = \left[\bar{v}_{\sigma_2}^{\,r_2}(p_2)\chi^{\mu}_{r_2 r_1}(s,t,u)u_{\sigma_1}^{\,r_1}(p_1)\right]\xi_\mu^{\,(\beta)}(d)\varphi_\pi, \qquad (1)$$

where $u_{\sigma_1}^{\,r_1}(p_1) \equiv u_1$ and $\bar{v}_{\sigma_2}^{\,r_2}(p_2) \equiv \bar{v}_2$ are the spinor and anti-spinor of the initial nucleons with spin projections σ_1 and σ_2 and dirac indices r_1 and r_2, respectively; $\xi_\mu(d)$ is the deuteron polarization vector, φ_π is the π-meson field; s, t, u are the invariant Mandelstam variables:

$$s = (p_1 + p_2)^2 \quad ; \quad t = (d - p_2)^2 \quad ; \quad u = (d - p_1)^2 \quad . \qquad (2)$$

This amplitude should be symmetrized over the initial nucleon states, and therefore it takes the form:

$$\bar{\mathcal{M}}^{\beta}_{\sigma_2,\sigma_1} = \frac{1}{\sqrt{2}}\left[\mathcal{M}^{\beta}_{\sigma_2,\sigma_1}(s,t,u) + (-1)^{\beta}\mathcal{M}^{\beta}_{\sigma_1,\sigma_2}(s,u,t)\right] \qquad (3)$$

The second term in (3), corresponding to the exchange of two nucleons, is equivalent to the exchange of the $t-$ and $u-$ variables.

Using transformation properties of the wave functions, one can finds the transformation lows of the spinor amplitude $\chi^{\mu}_{r_2 r_1}$. *The Lorenz-invariance* of matrix element under

the Lorenz transformation of all four-vectors $p' = \Lambda(\mathcal{A})p$ leads to the following Lorenz transformation low of the spinor amplitude:

$$\chi^{\mu}_{\alpha\beta}(p_1, p_2; d, \pi) = \mathcal{S}_{\alpha}^{\ \alpha'}(\mathcal{A})\chi^{\mu'}_{\alpha'\beta'}(p'_1, p'_2; d', \pi')\mathcal{S}_{\beta}^{\ \beta'}(\mathcal{A}^{-1})\Lambda_{\mu'}^{\ \mu}(\mathcal{A}^{-1}) \ . \tag{4}$$

With respect to discrete symmetries, we have from *P-invariance*

$$\chi^{\mu}(\vec{p}_1, \vec{p}_2; \vec{d}, \vec{\pi}) = \eta_{\mathcal{P}}\gamma_0\chi^{\mu}(-\vec{p}_1, -\vec{p}_2; -\vec{d}, -\vec{\pi})\gamma_0 g^{\mu\mu} \ , \tag{5}$$

where $\eta_{\mathcal{P}} = \frac{\eta_1\eta_2}{\eta_\pi\eta_d}(-1)^{s_d - s_1 - s_2} = (-1)$; η_i, s_i- are internal parities and spins of particles.

Time-reversal symmetry leads to time-reversal spinor amplitude $\chi_{\mu}^{\alpha\beta}$

$$\chi^{\mu}_{\alpha\beta}(\vec{p}_1, \vec{p}_2; \vec{d}, \vec{\pi}) = \eta_{\mathcal{T}}\mathcal{T}^{-1}_{\alpha\alpha'}\chi_{\mu}^{\alpha'\beta'}(-\vec{p}_2, -\vec{p}_1; -\vec{\pi}, -\vec{d})\mathcal{T}_{\beta'\beta} g^{\mu\mu} \ , \tag{6}$$

where the time-reversal matrix $\mathcal{T} = -i\gamma_5\mathcal{C}$.

The charge conjugation describe the connection of the spinor amplitudes χ for the process $NN \to d\pi$ and $\chi_{\mathcal{C}}$ for the charge conjugation process $\bar{N}\bar{N} \to \bar{d}\pi$:

$$\chi(\vec{p}_1, \vec{p}_2; \vec{d}, \vec{\pi}) = \eta_{\mathcal{C}}\mathcal{C}\chi^{t}_{\mathcal{C}}(\vec{p}_1, \vec{p}_2; \vec{d}, \vec{\pi})\mathcal{C}^{-1} \ . \tag{7}$$

The amplitude χ_μ for the process $NN \to d\pi$ can be expanded over six independent covariants, which can choice in such way that every of them satisfy the above properties. For this one we introduce the orthogonal system of four-vectors, one of them, P, is time-like, and other, p, N and L are space-like:

$$P = p_1 + p_2, \ p = (p_1 - p_2)/2, \ N_\mu = \varepsilon_\mu(p'pP), \ L_\mu = \varepsilon_\mu(NpP) \ . \tag{8}$$

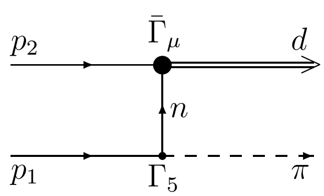
Here the four-vector $p' = (d - \pi)/2$. Then, one can get the whole system of orthogonal unit four-vectors $\{e^{(\sigma)}_{\mu}\}^{3}_{\sigma=0}$. Therefore, the spinor amplitude χ_μ can be expanded over this unit orthogonal system

$$\boxed{\chi_\mu = \chi_i e^{(i)}_\mu = \chi_1 l_\mu + \chi_2 n_\mu + \chi_3 e_\mu \ , \quad \chi_i = -\chi^\mu e^{(i)}_\mu \ = \gamma_5\left(a_i + b_i\hat{l}\right)} \tag{9}$$

3 Reaction Mechanism

- *One-nucleon exchange (ONE) and πNN-vertex.*

Within the framework of the one-nucleon exchange model, the amplitude χ_μ can be written in a simple form:



$$\chi_\mu = g^{+}\bar{\Gamma}_\mu(d)\mathcal{S}_{\mathcal{F}}(n)\Gamma_5(n) \ , \tag{10}$$

where $\bar{\Gamma}_\mu(d)$ is the deuteron vertex $pn \to d$ with one off-mass shell nucleon, $\mathcal{S}_{\mathcal{F}}(n) = (\hat{n} - m + i0)^{-1}$ is the fermion propagator and the value of the coupling constant is $g^{+} = \sqrt{2}g$, $g^2/4\pi = 14.7$. The vertex $\bar{\Gamma}_\mu(d)$ can be related to the deuteron wave function ($\mathcal{DWF}$) with the help of the following equation [9]:

$$\bar{\Psi}_\mu = \frac{\bar{\Gamma}_\mu}{n^2 - m^2 + i0} = \varphi_1(t)\gamma_\mu + \varphi_2(t)\frac{n_\mu}{m} + \left(\varphi_3(t)\gamma_\mu + \varphi_4(t)\frac{n_\mu}{m}\right)\frac{\hat{n} - m}{m} \ . \tag{11}$$

The form factors $\varphi_i(t)$ are related to two large components of the $\mathcal{DWF}$ u and w (corresponding to the ${}^3\mathcal{S}_1$ and ${}^3\mathcal{D}_1$ states) and to small components v_t and v_s (corresponding to the ${}^3\mathcal{P}_1$ and ${}^1\mathcal{P}_1$ states) as in [6].

In the theoretical description of processes at intermediate energies, the structure of hadrons is often described by multiplying the point-like operators by form factors. It is common practice to assume that these vertices, i.e. their operator structures and the associated form factors, are in all situations the same as for a free on-shell hadrons. In our case, however, the pion vertex can have a much richer structure: there can be more independent vertex operators and the form factors can depend on more than one scalar variable. The situation is similar to the construction of the off-shell electromagnetic vertex [10]. The common treatment of such off-shell effects is to presume them small and to ignore them by using the free vertices. However, as much of the present effort in intermediate energy physics focuses on delicate effects, such as evidence of quark/gluon degrees of freedom or small components in the wave functions, it is mandatory to examine these issues in detail [11].

The most general pion-nucleon vertex, where the incoming nucleon of mass m has momentum p_i^μ, the outgoing nucleon has momentum p_f^μ and the pion has momentum $\pi^\mu = p_f^\mu - p_i^\mu$, can be written as [12]

$$\Gamma_5\left(p_f, p_i\right) = \gamma_5 G_1 + \frac{\widehat{p}_f - m}{m}\gamma_5 G_2 + \gamma_5 \frac{\widehat{p}_i - m}{m} G_3 + \frac{\widehat{p}_f - m}{m}\gamma_5 \frac{\widehat{p}_i - m}{m} G_4 \; ; \tag{12}$$

here $\{G_i(t;p_i^2,p_f^2)\}_{i=1}^4$ are some functions depending on the relativistic invariant transfer $t=(p_i-p_f)^2$ and particles masses $p_{i,f}^2$ or the so-called pion form factors. By sandwiching Γ_5 between on-shell spinors one obtains $G_1(t,m^2,m^2)\bar{u}(p_f)\gamma_5 u(p_i)$.

In our case, one nucleon is the off-shell only, and therefore we will consider the "half-off-shell" vertex with incoming nucleon on-shell. We obtain in that case two terms in eq.(12) instead of four because the third and the fourth ones are vanishing, taking into account the Dirac equation for a free fermion. Then, eq.(12) can be written in the form:

$$\Gamma_5(t) = \gamma_5 \left(G_1(t) + G_2(t) \frac{\widehat{n} + m}{m} \right) \;= \lambda G^{\mathcal{PS}}(t)\gamma_5 + (1-\lambda)\, G^{\mathcal{PV}}(t) \frac{\widehat{\pi}}{2m}\gamma_5 \;, \tag{13}$$

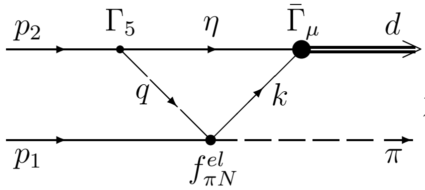
Note, according to the so-called equivalence theorem [13] the sum of all Born graphs for elementary processes, for example the pion photoproduction on a nucleon [14] and the other ones, is invariant under chiral transformation [15]. This means that starting with the Lagrangian appropriate to the pseudoscalar ($\mathcal{PV}$) coupling, one ends up in the Lagrangian appropriate to the pseudoscalar ($\mathcal{PS}$) coupling by performing a chiral transformation. This equivalence theorem is related to the processes for elementary particles. But in our case, for the reaction $NN \to d\pi$ there is a bound state, a deuteron, and therefore reducing this process to the one where only elementary particles participate, we will have the diagrams of a higher order over the coupling constant than the Born graph. So, the equivalence theorem cannot be applied to our considered processes. Therefore, the vertex Γ_5 in our case can be written in the form of eq.(13) which is actually a linear combination of pseudoscalar and pseudovector coupling with the so-called mixing parameter λ.

The dNN vertex has been studied by Buck and Gross [6] within the framework of the Gross equation of nucleon-nucleon scattering. They used a one boson exchange (OBE) model with π, ρ, ω and σ exchange. In their study, they suggest that the form factors $G^{\mathcal{PS}}$ and $G^{\mathcal{PV}}$ have the same t - dependence, in particular $G^{\mathcal{PS}}(t) = G^{\mathcal{PV}}(t) = h_N(t)$, and

consider $\lambda = 0.0; 0.2; 0.4; 0.6; 0.8$ and 1.0. In each case, the parameters of the OBE model were adjusted to reproduce the static properties of the deuteron. They found that the total probability of the small components of the $\mathcal{DWF}$: $P_{small} = \int_0^\infty p^2 dp\, [v_t^2(p) + v_s^2(p)]$, increases monotonically with growing λ from approximately 0.03% for $\lambda = 0$ to approximately 1.5% for $\lambda = 1$.

- *Second-order graphs*

Let us consider now the second order graph corresponding to the rescattering of the virtual π-meson by the initial nucleon. This mechanism of the $NN \to \pi d$ process has been analyzed by many authors, see, for example, [3, 4]. Our procedure of the construction of the helicity amplitudes corresponding to the triangle graph is similar to the ones published by [4], and so we present it briefly. The most important result of this integration is the nucleon spectator contribution where the nucleon labelled η is on mass shell ($\eta^2 = m^2$):



$$\chi_\mu^{sp} = \frac{g^+}{(2\pi)^3} \int h_\pi(q^2) \frac{\mathcal{F}_\mu\left(\vec{\eta}, \eta_0 = \sqrt{\vec{\eta}^2 + m^2}\right)}{q^2 - \mu^2} \frac{d^3\eta}{2\eta_0} \qquad (14)$$

where $h_\pi(q^2)$ is the pion form factor corresponding to the off-mass shell π-meson in the intermediate state; a monopole form has been chosen $h_\pi(q^2) = (\Lambda^2 - \mu^2)/(\Lambda^2 - q^2)$ as like as in [17]; here Λ is the corresponding cut-off parameter. The general form of $\mathcal{F}_\mu$ can be written as follows:

$$\mathcal{F}_\mu = \Gamma_5 \mathcal{S}^c_{\mathcal{F}}(\eta) \bar{\Gamma}_\mu(d) \mathcal{S}_{\mathcal{F}}(k) f^{el}_{\pi N} \ , \qquad (15)$$

where $f^{el}_{\pi N}$ is the amplitude of πN elastic scattering; it can be presented as expansion over two off-shell invariant amplitudes $f^{el}_{\pi N} = (A + B\widehat{\pi})$ which depend on four momenta. We compute A and B from the on-shell πN partial wave amplitudes $\mathcal{T}^{on}_{l\pm}(s_{\pi N})$ under the assumption

$$\mathcal{T}_{l\pm}(s_{\pi N}, t_{\pi N}, u_{\pi N}) \approx \mathcal{T}^{on}_{l\pm}(s_{\pi N}) \ , \qquad (16)$$

where $\mathcal{T}^{on}_{l\pm}(s_{\pi N})$ are taken from the Karlsruhe-Helsinki phase shift analysis [18]. However, in the partial wave decomposition of the invariant functions, full off-shell angular momentum projectors are used for the lowest waves in the manner discussed for the $NN \to NN\pi$ reaction in Ref.[19].

The triple integral (14) over azimuth φ_η, polar angle ϑ_η and the magnitude of three-momentum η must be done numerically for which we used a Gaussian quadrature. There are 6 triple integrals over a complicated complex integrand for each scattering angle.

4 Results and Discussions

In order to investigate the effect of small components of the $\mathcal{DWF}$, we have calculated the differential cross section $d\sigma/d\Omega$, polarization characteristics A_{ii}, A_{y0}, etc. for $pp \to d\pi^+$ as a function of scattering angle at proton kinetic energy $T_p = 578 MeV$ corresponding to pion kinetic one $T_\pi = 147 MeV$ because at this energy the probability of Δ-isobar production by the two - step mechanism is rather sizeable. All the calculated quantities

are in the Madison convention and compared with the experimental data [20] and partial-wave analysis ($\mathcal{PWA}$) by R. A. Arndt et al. [21] (dotted curve). The cut-off parameter Λ and the mixing one λ corresponding to the πNN vertex are chosen by the best fitting of the experimental cross section $d\sigma/d\Omega$ data. We have checked that the polarization curves change very little if we vary the cut-off parameter Λ.

Note that the contribution of the triangle graph is very large at intermediate initial kinetic energies and much smaller at lower energies. It is caused by a large value of the cross section of elastic πN scattering because of a possible creation of the Δ-isobar at this energy. One can stress that the application of Locher's form $\mathcal{DWF}$ [4] does not allow one to reproduce the absolute value of the differential cross section (see Fig. 1.) over the whole region of scattering angle ϑ. But using the Gross approach for the $\mathcal{DWF}$ [6], one can describe $d\sigma/d\Omega$ at $\lambda = 0.6 - 0.8$ rather well.

The next interesting result which can be seen from Fig. (2-6) is a large sensitivity of all the polarization characteristics to the small components of the $\mathcal{DWF}$. The asymmetry A_{y0} (Fig. 2.) and the vector polarization iT_{11} (Fig. 3.) calculated within the framework of Gross's approach particularly show this large sensitivity. These quantities are interference dominated and sensitive to the phases. The results for iT_{11} have a wrong sign with Locher's form $\mathcal{DWF}$ [4]. On closer inspection, we observe that the first term in eq.(21), $(\Phi_1^* - \Phi_3^*)\Phi_2$, is very big due to constructive interference $\Phi_1 \approx -\Phi_3$. It is caused by the $N\triangle$ configuration in a relative $\mathcal{S}$ wave having pp spin zero ($^1\mathcal{D}_2$ state). The $^1\mathcal{D}_2$ partial-wave dominates making $\Phi_{1,2,3}$ large, but the results are the same contribution to $\Phi_1^{\mathcal{J}=2}$ and $\Phi_3^{\mathcal{J}=2}$ (with opposite signs caused by the relevant Wigner d-function signature). Since the contribution of $\Phi_{4,5,6}$ is negligible, the sign problem for iT_{11} is therefore very sensitive to the $\Phi_2^{\mathcal{J}=0}$ (or $^1\mathcal{S}_0$) partial wave. As iT_{11} is very nearly proportional to Φ_2, the phase of Φ_2 determines the sign of iT_{11}.

The right structure of the observables starts to appear gradually in the theoretical curves as one increases the mixing parameter λ in the Buck-Gross model, that is to say, as one increases the probability of the small components in the $\mathcal{DWF}$. We have checked that this structure originates indeed from the small components v_t and v_s in eq.(11). If we make $v_t = v_s = 0$ in the Buck-Gross model, then all curves become very similar to Locher's ones. Similarly, if we vary the πNN vertex given by eq.(13) by considering λ between 0 and 1 but keep Locher's $\mathcal{DWF}$, then the curves change very little again.

The proton spin correlations A_{ii} are presented in Fig. (4-6). Actually, the data on A_{zz} (Fig. 4.) is the measure of the $\Phi_{4,5,6}$ magnitudes because the deviation of A_{zz} from -1 is determined by these amplitudes (20). According to the partial wave decomposition, Φ_4 and Φ_6 are the amplitudes containing only triplet spin states in the pp channel. One can conclude that the magnitudes of the spin-triplet amplitudes are somewhat small. As for A_{yy} (Fig. 5.) and A_{xx} (Fig. 6.), the terms proportional to $\Phi_1 + \Phi_3$ can be neglected because there is a phase relation $\Phi_1 \approx -\Phi_3$. Therefore, the deviation of A_{yy} and A_{xx} from -1 is determined by $\Phi_{4,6}$ again, whereas Φ_5 does not contribute to the numerator of A_{yy}.

One can also see a large sensitivity of the observables A_{ii} to the used form of $\mathcal{DWF}$. The application of Gross's approach by the construction of $\mathcal{DWF}$ [6] results in the shapes of these characteristics which are different from the corresponding ones obtained within the framework of Locher's approach [4].

Note, the energy dependence of all the observables within the framework of the suggested approach is the subject of our next investigation.

5 Summary and Outlook

A relativistic model for the reaction $NN \to d\pi$ has been discussed in detail using two forms of the $\mathcal{DWF}$ [4] and [6]. One of them [4] was already used in the analysis of the $pp \to d\pi$ process also taking into account the two-step mechanism with a virtual pion in the intermediate state. The difference between our approach and the model considered in [4] is the following. We have analyzed the sensitivity of all the observables to the form of $\pi NN-$current and the choice of the $\mathcal{DWF}$ relativistic form. First of all, from the results presented in Fig. (1-6), one can see very large sensitivity of all the observables, especially of the polarization characteristics to the choice of the $\mathcal{DWF}$ form. The inclusion of the P-wave contribution in the $\mathcal{DWF}$ within the framework of Gross's approach [6] results in a better description of the experimental data on the differential cross section and the polarization observables. The next interesting result is related to the extraction of some new information on the off-shell effects due to a virtual (off-shell) nucleon. Comparing the observable with the experimental data (see Fig. (1-6)), one can test the assumption, suggested by [6], of a possible form of the pion form factor and conclude that one cannot use the mixing parameter $\lambda = 1$ as like as in [4].

One can stress that the one-nucleon exchange and the pion rescattering graphs have been studied only in this paper in order to investigate very important effects: off-mass shellness of nucleon and pion, and $P-$wave contribution to the $\mathcal{DWF}$. The interactions in the initial NN and final $d\pi$ states can be in principle contributed to the total amplitude of the considered reaction. However, it will be as a separate stage of this study because a more careful inclusion of elastic NN and $d\pi$ interactions at intermediate energies is needed.

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6 Appendix.

- *Helicity formalism.*

To calculate the observables, differential cross sections and polarization characteristics, it would be very helpful to construct the helicity amplitudes of the considered process $NN \to d\pi$. So, we use the helicity formalism for this reaction presented in Ref.[8].

Let us introduce initial nucleon helicities μ_1, μ_2 and the final deuteron λ, and helicity amplitudes $\bar{\mathcal{M}}^{\lambda}_{\mu_2,\mu_1}(W,\vartheta)$ depending on the initial energy W in the $N-N$ c.m.s. and the scattering angle ϑ analogous to [4]. This amplitude $\bar{\mathcal{M}}^{\lambda}_{\mu_2,\mu_1}(W,\vartheta)$ corresponds to the transition of the NN system from the state with helicities $\mu_1, \mu_2 = \pm 1/2$ to the state with $\lambda = \pm 1, 0$.

With respect to discrete symmetries, we have from *parity conservation* (5):

$$\mathcal{M}^{\lambda}_{\mu_2\mu_1} = \eta_P(-1)^{(\mu_2-\mu_1)-\lambda}\mathcal{M}^{-\lambda}_{-\mu_2-\mu_1} = (-1)^{\mu_2+\mu_1+\lambda}\mathcal{M}^{-\lambda}_{-\mu_2-\mu_1} \ . \tag{17}$$

Time - reversal symmetry (6) leads to

$$\mathcal{M}^{\lambda}_{\mu_2\mu_1} = (-1)^{(\mu_2-\mu_1)-\lambda}\mathcal{M}^{\mu_2\mu_1}_{\lambda} \ . \tag{18}$$

We use the abbreviations for helicity amplitudes as [5]. Using the expansion (9), one can get the following form of the helicity amplitudes:

$$\Phi_{\substack{1\\3}} = \bar{\mathcal{M}}^{\pm}_{++} = \mp\frac{1}{\sqrt{2}}\frac{\varepsilon}{m}\Big[a_1^s\cos\vartheta \pm ia_2^a - a_3^a\sin\vartheta\Big],\ \Phi_2 = \bar{\mathcal{M}}^{0}_{++} = \frac{\varepsilon\varepsilon_d}{mM}\Big[a_1^s\sin\vartheta + a_3^a\cos\vartheta\Big],$$

$$\Phi_{\substack{4\\6}} = \bar{\mathcal{M}}^{\pm}_{+-} = \mp\frac{1}{\sqrt{2}}\frac{p}{m}\Big[b_1^s\cos\vartheta \pm ib_2^s - b_3^a\sin\vartheta\Big],\ \Phi_5 = \bar{\mathcal{M}}^{0}_{+-} = \frac{p\varepsilon_d}{mM}\Big[b_1^s\sin\vartheta + b_3^a\cos\vartheta\Big], \quad (19)$$

where $\chi_i^{\{^s_a\}}(s,t,u)$ are symmetric and antisymmetric combinations $\chi^{\{^s_a\}} = (\chi_i(\vartheta) \pm \chi_i(\pi - \vartheta))/\sqrt{2}$. All symmetry properties (17) are satisfied by these amplitudes.

- *Observables.*

Using the helicity amplitudes (19), one can calculate the all observables: differential cross section, asymmetry, deuteron tensor polarization and so on. Let us present now the expressions for the following observables in the c.m.s. using Φ_i:

$$\begin{aligned}
A_{y0} &= 4Im(\Phi_1\Phi_4^* + \Phi_2\Phi_5^* + \Phi_3\Phi_6^*)\Sigma^{-1}, \quad A_{0y}(\theta) = A_{y0}(\pi-\theta),\\
A_{xz} &= -4Re(\Phi_1\Phi_4^* + \Phi_2\Phi_5^* + \Phi_3\Phi_6^*)\Sigma^{-1}, \quad A_{zx}(\theta) = A_{xz}(\pi-\theta),\\
A_{zz} &= -1 + 4(|\Phi_4|^2 + |\Phi_5|^2 + |\Phi_6|^2)\Sigma^{-1},\\
A_{yy} &= -1 + 2(|\Phi_1+\Phi_3|^2 + |\Phi_4+\Phi_6|^2)\Sigma^{-1},\\
A_{xx} &= A_{zz} + 2(|\Phi_1+\Phi_3|^2 - |\Phi_4+\Phi_6|^2)\Sigma^{-1}.
\end{aligned} \quad (20)$$

The expressions for the deuteron tensor polarization components are the following:

$$\begin{aligned}
iT_{11} &= -\sqrt{6}Im\left[(\Phi_1^* - \Phi_3^*)\Phi_2 + (\Phi_4^* - \Phi_6^*)\Phi_5\right]\Sigma^{-1},\\
T_{20} &= \Big[1 - 6(|\Phi_2|^2 + |\Phi_5|^2)\Sigma^{-1}\Big]/\sqrt{2},\\
T_{21} &= \sqrt{6}Re\left[(\Phi_1^* - \Phi_3^*)\Phi_2 + (\Phi_4^* - \Phi_6^*)\Phi_5\right]\Sigma^{-1},\\
T_{22} &= 2\sqrt{3}Re(\Phi_1^*\Phi_3 + \Phi_4^*\Phi_6)\Sigma^{-1} = (1 + 3A_{yy} - \sqrt{2}T_{20})/(2\sqrt{3})\ .
\end{aligned} \quad (21)$$

The variable Σ is related to the differential cross section as

$$\Sigma = 2\sum_1^6 |\ \Phi_i\ |^2 = 4\frac{p}{k}\left(\frac{m}{4\pi\sqrt{s}}\right)^{-2}\frac{d\sigma}{d\Omega} = \frac{1}{\sigma_0}\frac{d\sigma}{d\Omega}\ , \quad (22)$$

where p and k are the momenta of initial proton and final deuteron in the c.m.s.

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[21] Those with access to TELNET can run the SAID program with a link to http://clsaid.phys.vt.edu

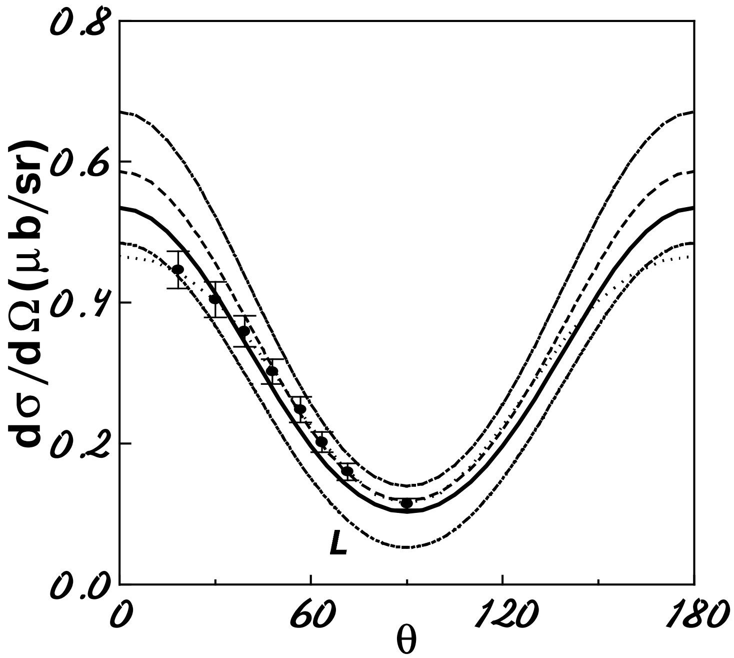

Figure 1: Differential cross section $d\sigma/d\Omega$ for $pp \to d\pi^+$ as a function of scattering angle in the c.m.s. at $T_p = 578MeV$ when the cut-off parameter Λ and mixing one λ varied simultaneously both in the deuteron wave function and in the πNN vertex. The dashed ($\lambda = 0.6; \Lambda = 1$), solid ($\lambda = 0.8; \Lambda = 0.6$) and dot-dashed ($\lambda = 1; \Lambda = 0.6$) lines correspond to the Gross $\mathcal{WFD}$ [6]. The dot-dot-dashed line corresponds to the results with Locher's $\mathcal{WFD}$ [4] ($\lambda = 1; \Lambda = 1$). The dots represent the partial-wave analysis by R. A. Arndt et al. [20]. The data are from [4, 20]. All spin observables are in the Madison convention.

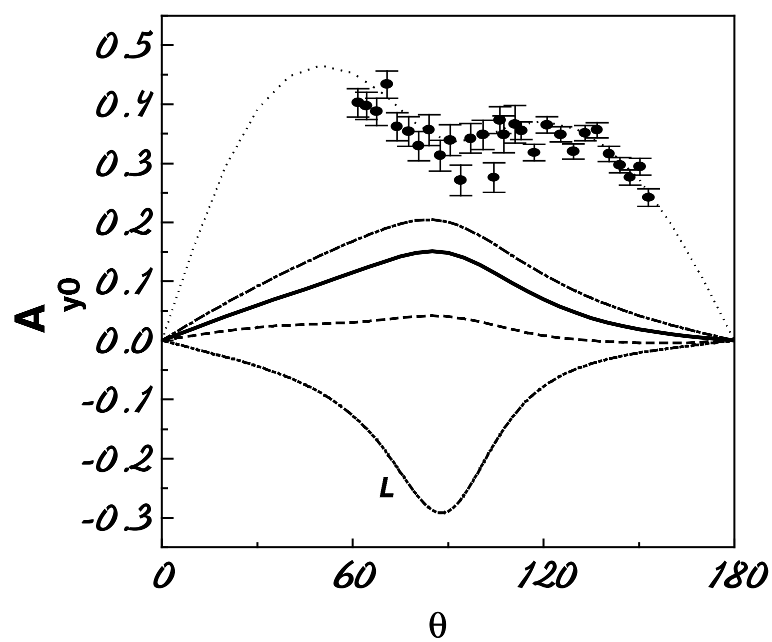

Figure 2: Assymetry A_{y0}. Notation as in Fig. 1.

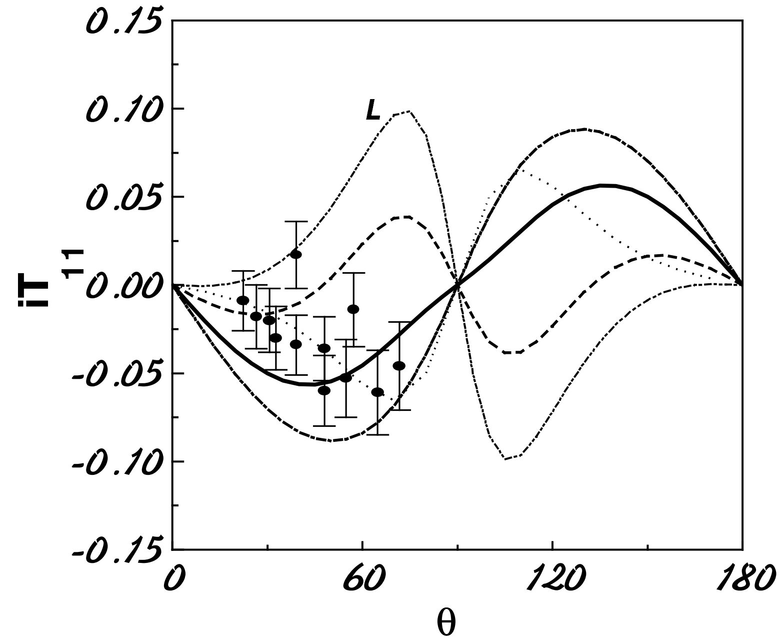

Figure 3: Vector polarization iT_{11}. Notation as in Fig. 1.

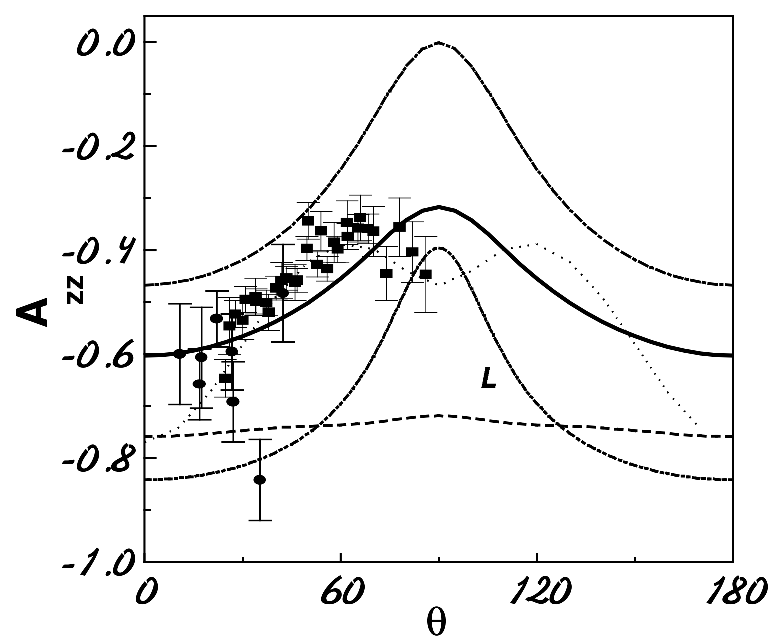

Figure 4: Spin correlation A_{zz}. Notation as in Fig. 1.

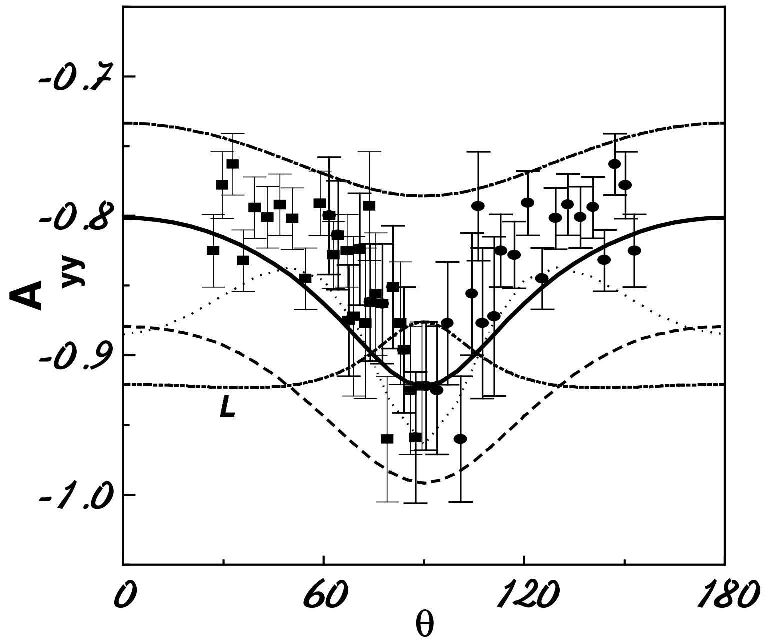

Figure 5: Spin correlation A_{yy}. Notation as in Fig. 1.

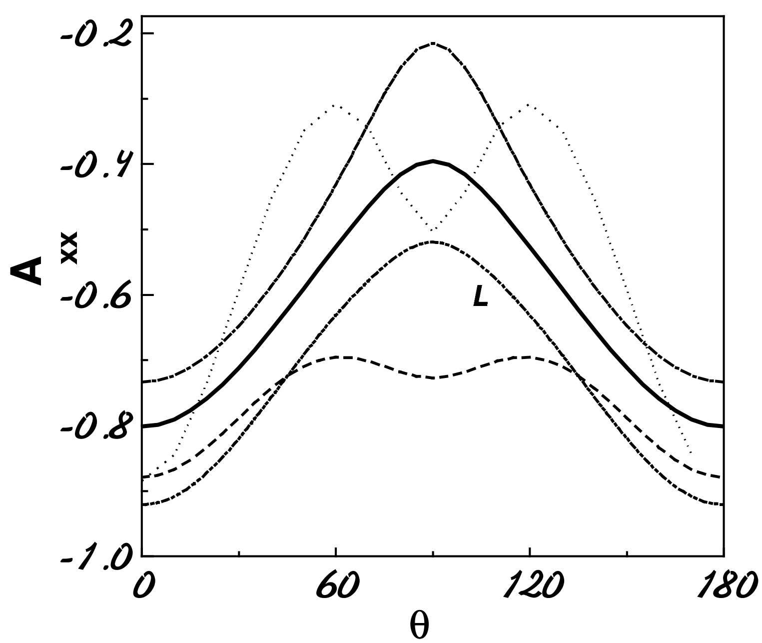

Figure 6: Spin correlation A_{xx}. Notation as in Fig. 1.